Reprinted from Journal of Applied Physics, Vol. 36, No. 11, 3655–3659, November

Theoretical Approach to Enhanced Pressure Apparatus Design*

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(Received 28 June 1965)

The possibility of obtaining an enhanced stress level within a solid is investigated for a spherical configuration consisting of a solid homogeneous isotropic core encapsulated by a nonhomogeneous isotropic shell. Utilizing a linear elastic analysis, the enhancement is maximized (1) for the case when the nonhomogeneous shell is composed of a number of concentric homogeneous shells; and (2) for the case where the elastic shell modulus varies as a power of the radius. The dependence of the enhancement on the various parameters is depicted graphically and the results are discussed in terms of presently available materials. It is felt that the attainable enhancement may be significant only in the region up to 10 kbar and very limited above 30 kbar.

INTRODUCTION

THE actual state of stress within a solid body subjected to external stresses is often the topic of lively discussion among personnel involved in high-pressure research. If the solid body is a single crystal or a truly isotropic, homogeneous material the situation is well understood. On the other hand, if the solid body is of a nonhomogeneous nature such as that encountered in polycrystalline material, during phase transitions, or when dealing with a composite body, the situation is more complex. The possibility of large stress variations within a composite solid body (specifically an encapsulated sample) was pointed out by Bobrowsky[1] in connection with interpretation of pressure measurements and calibrations. If this composite body consists of a solid-core sample which is more compressible than the encapsulating material, the results are intuitive. The resulting stress level in the sample is less than the externally applied pressure. A less intuitive situation is the inverse in which the encapsulating material has a higher compressibility than the solid-core sample. In this case the resulting stress level in the sample may be considerably higher than the externally applied pressure. During most high-pressure research there is an effort to eliminate, minimize, or at least standardize these effects.

Alternatively this paper investigates the possibility of capitalizing on these effects to expand the usefulness of present apparatus. The encapsulation may be considered to act as an intensifier or enhancer, thus two-staging present apparatus. The following sections predict how large this pressure enhancement may be and postulate under what conditions constructing such a device would be a worthwhile investment of time and effort.

ANALYSIS AND RESULTS

A spherical configuration with its complete geometrical symmetry would appear to be the most suitable choice for obtaining the desired hydrostatic stress in the solid core. This configuration, shown in Fig. 1, consists of a solid isotropic core of radius a surrounded by a nonhomogeneous isotropic encapsulating shell of inner radius a and outer radius b. The ratio of the bulk moduli of the shell and core material is restricted to values felt to be compatible with present materials and the ratio of the radii (a/b) is restricted to values which yield a reasonable sample size if the configuration were to be placed in the sample chamber of presently available 30-kbar liquid-medium pressure apparatus. Poisson's ratio, though having a limited range for real materials, is allowed to vary to its extremes for the sake of illustration. The analysis assumes the materials to be in the elastic state and no attempt is made to account for variation of moduli with stress. The results, however, depend on ratios of the moduli rather than the moduli themselves and should be applicable to higher pressures where the moduli increase but the ratios may remain relatively constant.

The external surface of the shell is subjected to a hydrostatic pressure p_e which produces a hydrostatic

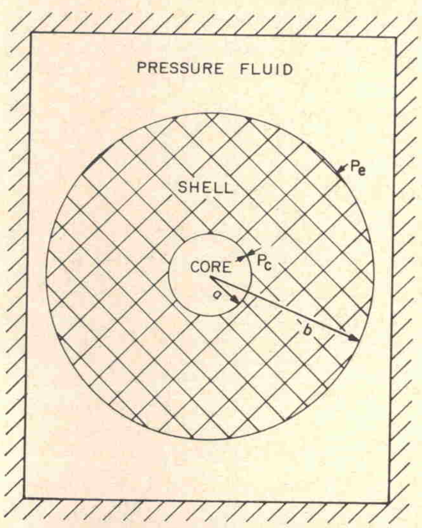

Fig. 1. Cross section of the spherical configuration investigated. The external hydrostatic stress on the configuration is supplied by the pressure fluid of a conventional pressure apparatus.

*Work supported by the U. S. Atomic Energy Commission.
[1] A. Bobrowsky in *High-Pressure Measurement* (Butterworths, Inc., Washington, D. C., 1963), p. 172.

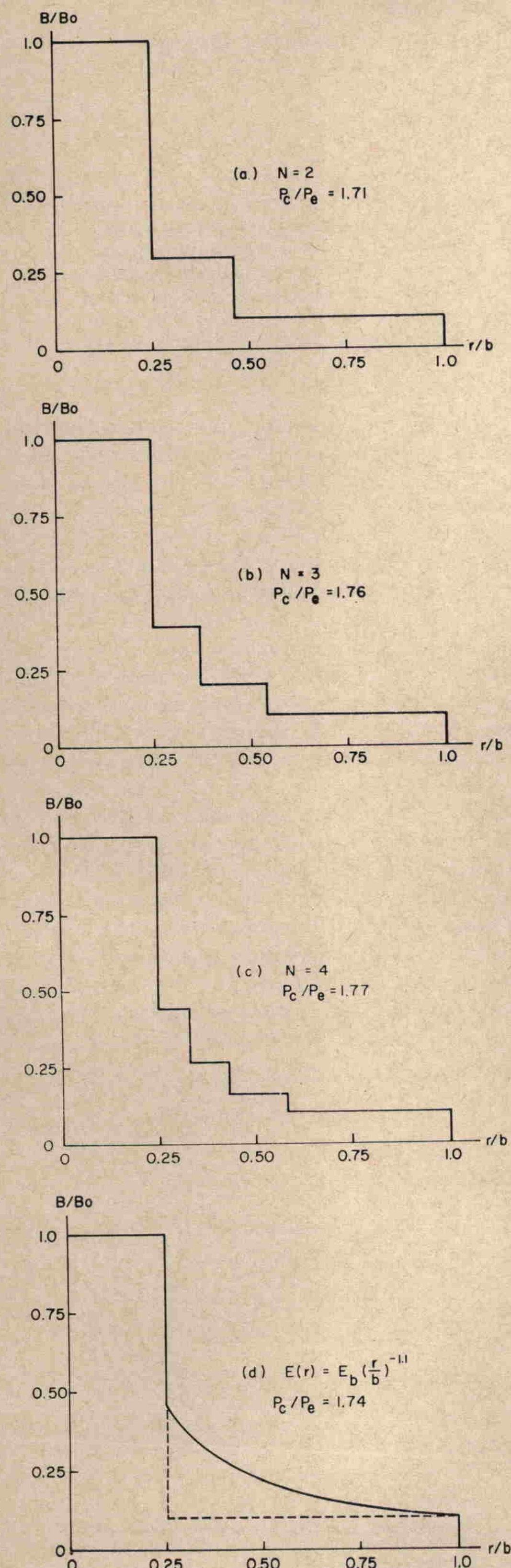

FIG. 2. Variation of the bulk modulus with radius for maximum pressure ratio (p_c/p_e). $\kappa=0.615$, $(a/b)=\frac{1}{4}$, $(B_b/B_0)=\frac{1}{10}$.

pressure p_c on the elastic core. The ratio of the pressures p_c/p_e is designated as the pressure enhancement ratio and it is desired to determine the shell nonhomogeneity so as to maximize this ratio. It is assumed that in the nonhomogeneous shell, the elastic modulus E varies with radius r and that Poisson's ratio ν is constant. This assumption is in line with previous investigations of nonhomogeneous elastic media.[2–4]

Subject to the stated conditions, the deformation of the shell is completely symmetrical and described by the radial displacement $u=u(r)$. The radial and tangential stresses associated with this deformation are given by[5]

$$\sigma_r=[E(r)/(1+\nu)(1-2\nu)]\times[(1-\nu)(du/dr)+2\nu(u/r)], \quad (1)$$
$$\sigma_t=[E(r)/(1+\nu)(1-2\nu)][\nu(du/dr)+(u/r)].$$

Equilibrium in the radial direction requires

$$(d\sigma_r/dr)+(2/r)(\sigma_r-\sigma_t)=0, \quad (2)$$

which, upon substitution from (1) renders

$$E(r)[(d^2u/dr^2)+(2/r)(du/dr)-2(u/r^2)] +(dE/dr)\{(du/dr)+[2\nu/(1-\nu)](u/r)\}=0. \quad (3)$$

Formally, the problem consists of determining the function $E(r)$ such that the pressure ratio p_c/p_e is a maximum, subject to the conditions that the ratios (a/b) and $[E(b)/E(a)]$ are fixed. In the opinion of the authors, the resulting variational problem is complicated to a degree which precludes a formal solution for $E(r)$.

Alternatively, it is illuminating to consider the nonhomogeneous shell as made up of N homogeneous concentric shells, the ith shell occupying the region $r_i\leq r\leq r_{i+1}$, where $r_1=a$ and $r_{N+1}=b$. The modulus of elasticity is constant in the ith shell and designated simply E_i. The quantity E_b represents the modulus at $r=b$. The displacement and radial stress in the ith shell is then representable in the form

$$\left.\begin{aligned} u_i&=c_ir+d_i/r^2\\ \sigma_{r_i}&=3B_i[c_i-\kappa d_i/r^3]\end{aligned}\right\} r_i\leq r\leq r_{i+1}, \quad (4)$$

where $B_i=E_i/3(1-2\nu)$ is the shell bulk modulus, $\kappa=2(1-2\nu)/(1+\nu)$, and c_i, d_i are constants. The quantity $B_b=E_b/3(1-2\nu)$ designates the shell bulk modulus at the radius $r=b$. The displacement and radial stress in the homogeneous solid core $0\leq r\leq a$ are given by

$$\left.\begin{aligned} u_0&=c_0r\\ \sigma_{r0}&=-p_c=3B_0c_0\end{aligned}\right\} 0\leq r\leq a, \quad (5)$$

where $B_0=E_0/3(1-2\nu_0)$ is the bulk modulus of the core material and c_0 is a constant. The boundary condi-

[2] K. Hruban, Bull. Intern. Acad. Tcheque. Sci. **46**, 151 (1945).
[3] Y. Chen, J. Franklin Inst. **275**, 88 (1963).
[4] J. Mahig, J. Appl. Mech. **31**, 343 (1964).
[5] S. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw-Hill Book Co., Inc., New York, 1951), 2nd ed., p. 417.